



Discovering and Applying the Pythagorean Theorem

GRADE	SUBJECT	DURATION	DATE
8	Mathematics	3 days – 60 min each	2026-07-05

Unit Overview

Day 1	Discovering the relationship between the areas of squares built on the sides of a right triangle to define the Pythagorean theorem.	60 min
Day 2	Calculating missing side lengths of right triangles, including both the hypotenuse and the legs, using algebraic steps and square roots.	60 min
Day 3	Modeling and solving real-world distance problems by drawing right triangles, calculating the unknown lengths, and explaining the mathematical reasoning.	60 min

STAGE 1 Learning Standards

KEY CONCEPTS

- The Pythagorean theorem describes a unique area relationship ($a^2 + b^2 = c^2$) that only exists among the three sides of a right triangle.
- Knowing any two side lengths of a right triangle allows us to calculate the exact length of the third side using algebraic operations and square roots.
- Real-world spatial relationships and diagonal distances can be modeled as right triangles to solve practical measurement problems.

ESSENTIAL SKILLS

- Draw and label a right triangle diagram to represent a written real-world scenario.
- Identify and label the legs and the hypotenuse of any right triangle, regardless of its orientation.
- Explain the geometric connection between the area of squares on each side of a right triangle and the theorem's formula.
- Estimate and calculate square roots to find precise decimal or whole-number side lengths.

LEARNING OBJECTIVES

- Show how the sum of the areas of the two smaller squares equals the area of the largest square in a right triangle.
- Solve for a missing hypotenuse (c) when given the lengths of both legs (a and b).
- Solve for a missing leg (a or b) when given the lengths of the hypotenuse and one leg.
- Write out the step-by-step mathematical reasoning used to solve a real-world distance problem.

CORE CONTENT

- Right triangle anatomy: identifying the right angle, legs (a and b), and the hypotenuse (c as the longest side opposite the right angle).
- The Pythagorean theorem formula:
 $a^2 + b^2 = c^2$.
- Perfect squares and estimating non-perfect square roots to the nearest tenth.
- Algebraic steps to isolate a variable when solving for a missing leg (such as $c^2 - a^2 = b^2$).
- Word problem strategies: identifying key terms that indicate vertical, horizontal, and diagonal distances.

INQUIRY QUESTIONS

- Why does the Pythagorean relationship only work for right-angled triangles and not acute or obtuse ones?
- How does drawing a picture help us see hidden right triangles in the world around us?
- When we calculate a missing side length, how can we check if our answer is realistic and makes sense?

STAGE 2 **Assessment Evidence**

SUMMATIVE TASKS

- The Backyard Design Portfolio: Students create a scale map of a backyard on grid paper that includes three distinct right-triangle features (such as a diagonal stone path, a wheelchair ramp, and a string of lights hanging from a post to a fence). For each feature, students must draw the right triangle, label the known legs or hypotenuse, write out the algebraic steps using $a^2 + b^2 = c^2$ to solve for the missing side length to the nearest tenth, and write a brief explanation of how they verified that their calculated distance is mathematically reasonable.

FORMATIVE CHECKS

- **Anatomy of a Triangle Exit Ticket:** At the end of Day 1, the teacher collects a quick exit ticket where students are given three right triangles in different orientations (tilted and upside down). Students must circle the right angle, highlight the hypotenuse, write down which sides are the legs, and write a one-sentence rule they use to always spot the hypotenuse.
- **Whiteboard Check-In and Error Analysis:** During Day 2 practice, the teacher displays a problem where a student made a common error (such as adding instead of subtracting when finding a missing leg: $10^2 + b^2 = 12^2$ leading to $b^2 = 100 + 144$). Students use individual whiteboards to find the mistake, write down the correct algebraic steps to isolate the leg, and hold up their boards for a quick teacher scan to identify who needs a small-group guided re-teach.
- **Drafting Triangles Peer Conference:** On Day 3, before starting the final task, students work in pairs to read three real-world word problems. Using a checklist, they must draw and label the right triangle for each scenario without solving them yet. The teacher circulates with a clipboard checklist to note which students struggle to correctly place the vertical height, horizontal distance, and diagonal hypotenuse based on the text clues.

SUCCESS CRITERIA (PROFICIENCY SCALE)

BEG

I can identify the legs and the hypotenuse on a labeled right triangle diagram and substitute those values into the formula $a^2 + b^2 = c^2$ with teacher prompting or visual guides.

DEV

I can independently solve for a missing hypotenuse (c) or a missing leg (a or b) when given a clear right triangle diagram with two whole-number side lengths, showing my algebraic steps.

PRO

I can read a real-world word problem, draw and label a corresponding right triangle diagram, and calculate the missing side length to the nearest tenth, explaining why my final answer makes sense.

ADV

I can analyze a complex real-world multi-step scenario (such as finding a diagonal distance across two connected right triangles), calculate the exact missing lengths, and write a clear mathematical justification explaining how a change in one leg's length impacts the overall diagonal distance.

STAGE 3 Learning Experiences & Instruction

Day 1 of 3 Discovering the Area Relationship

60 min

DESIGNING A SCREEN LAYOUT

Phone designers have to fit a rectangular screen into a pocket-sized case. When they increase the width and height of the screen, the diagonal length across the glass changes in a very specific, predictable way.

How do changes in flat screen dimensions affect diagonal screen space?

CUTTING ACROSS THE PARK

Walking along the paved sidewalk around a square park takes a lot longer than cutting straight across the grass from corner to corner. Taking this shortcut always saves you a consistent amount of steps.

Why does cutting diagonally across a space always save us steps?

ENGAGE **Sorting Triangles by Angles**

Teacher provides paper triangles of various shapes for students to sort into acute, obtuse, and right-angled groups to notice what makes right triangles unique.

Recommended Worksheets: Geometry Practice – Grades 6–8

EXPLORE **Building Squares on Sides**

Students use grid paper and square tiles to build physical squares on all three sides of several right triangles and count the total grid units in each square.

Recommended Worksheets: Mathematical Thinking – Grades 6–8; Coordinate Plane Practice – Grades 6–8

EXPLAIN **Connecting the Areas**

Teacher guides students to calculate the area of each square and write down the pattern they notice when adding the two smaller areas together.

Recommended Worksheets: Test & Exam Reflection – Grades 6–8; Cornell Notes – Grades 9–12; Learning Portfolio Reflection – Grades 6–8

I DO

Teacher demonstrates how to build a physical square on the hypotenuse of a 3–4–5 right triangle and calculates its area on the board.

WE DO

Teacher guides students to build squares on the legs of a 5–12–13 triangle and helps them add the two smaller areas together.

YOU DO

Students work in pairs to build squares on a new right triangle, calculate the areas, and write down the equation that represents their findings.

APPLY Testing the Rule

Students test this area sum rule on non-right triangles to see if the relationship still holds true or if it only applies to right angles.

Recommended Worksheets:

3-Act Math Task - Grades 6-8; Structured Algebra Problem Set - Grades 6-8; Computation Practice - Grades 6-8

REFLECT Defining the Theorem

Students write a summary in their own words explaining why the area relationship only works for right triangles.

Recommended Worksheets: Math Fact Fluency - Grades 6-8

Day 2 of 3 Calculating Missing Side Lengths

60 min

RECONNECT FROM DAY 1

We carry forward our understanding of the area relationship ($a^2 + b^2 = c^2$) from Day 1 to calculate exact side lengths using square roots and algebraic steps.

PROGRAMMING A ROBOT PATH

A robotic vacuum needs to travel from one corner of a room to the opposite corner. The programmer knows the exact length and width of the room but needs to code the diagonal path.

How can a computer calculate a diagonal path without measuring it?

BUILDING A SKATE RAMP

Skateboarders want to build a wooden ramp with a specific height and a flat base length. They need to buy a single piece of plywood for the slanted surface.

How can we determine the ramp length before buying the wood?

ENGAGE Estimating Square Roots

Students use a number line to estimate the values of non-perfect square roots to the nearest tenth before using a calculator.

Recommended Worksheets: Choice Board - Grades 9-12; Gratitude Journal - Grades 6-8

EXPLORE Solving for the Hypotenuse

Students use the formula to find the missing diagonal side when given both legs of a right triangle, showing each algebraic step.

Recommended Worksheets: Fractions Practice - Grades 6-8; Integers & the Number Line - Grades 6-8

EXPLAIN Isolating a Missing Leg

Teacher models the algebraic steps needed to subtract a leg's area from the hypotenuse's area to find a missing leg.

Recommended Worksheets: Concept Review – Grades 6–8

I DO

Teacher models how to solve for a missing leg by writing out the subtraction steps and estimating the final square root on a number line.

WE DO

Teacher guides students through a whiteboard practice problem where they must identify whether the missing side is a leg or the hypotenuse before solving.

YOU DO

Students independently solve a set of mixed problems finding both legs and hypotenuses, showing all algebraic steps and rounding to the nearest tenth.

APPLY Spotting the Error

Students analyze a solved problem containing a common mistake, highlight the error, and write out the correct steps on their whiteboards.

Recommended Worksheets: Error Analysis Task Cards

REFLECT My Solving Checklist

Students write a step-by-step checklist explaining how to decide whether to add or subtract when solving for a missing side.

Recommended Worksheets: Self-Correction Checklist

Day 3 of 3 Applying the Theorem to Real-World Problems

60 min

RECONNECT FROM DAY 2

We transfer our skills in calculating missing hypotenuses and legs from Day 2 to model, solve, and explain complex real-world distance problems.

SETTING UP A WI-FI NETWORK

Network technicians need to run a fiber-optic cable from a high utility pole directly to the corner of a nearby building. They must avoid underground pipes by running the cable diagonally through the air.

How do technicians calculate the cable length needed between two points?

PITCHING A CAMPING TENT

Campers want to secure their tent using a rope tied from the top of the tent pole to a metal stake in the ground. The rope must be pulled tight to keep the tent stable.

What determines how far from the tent the stake should go?

ENGAGE**Translating Words to Drawings**

Students read short descriptions of real-world scenarios and underline words that indicate vertical, horizontal, or diagonal lines.

Recommended Worksheets: Scenario Translation Sheet

EXPLORE**Drafting Right Triangles**

Students work in pairs to draw and label right triangles representing real-world situations, placing the right angle correctly based on text clues.

Recommended Worksheets: Drafting Triangles Checklist

EXPLAIN**Justifying the Distance**

Teacher demonstrates how to write a mathematical explanation that justifies why a calculated real-world distance is reasonable.

Recommended Worksheets: Written Justification Guide

I DO

Teacher models how to read a word problem about a ladder sliding down a wall, draw the diagram, solve for the missing leg, and write a justification.

WE DO

Teacher guides students to set up a diagram for a wheelchair ramp problem, helping them identify which values represent the rise, run, and ramp length.

YOU DO

Students independently complete their backyard design portfolio, drawing the triangles, calculating the missing distances, and writing their mathematical justifications.

APPLY**Creating the Backyard Map**

Students apply their skills to create a scale map of a backyard featuring three distinct right-triangle elements, calculating the missing distances.

Recommended Worksheets: Backyard Design Portfolio Template

REFLECT**Peer Feedback Gallery Walk**

Students review a partner's backyard map, check the calculations, and write one constructive comment about their mathematical reasoning.

Recommended Worksheets:

Peer Review Rubric; Triangle Sorting Cards; Grid Paper and Tile Activity Sheet; Area Relationship Anchor Chart; Triangle Testing Worksheet; Day 1 Exit Ticket; Square Root Estimation Guide; Hypotenuse Practice Guide; Algebraic Isolation Template

UNIT-LEVEL Learning Enhancements

EQUITY

- Ensure that real-world scenarios in the unit do not assume access to specific home environments or expensive hobbies, instead centering accessible community spaces and public infrastructure.
- Provide multiple entry points for the geometric proof by using tactile square tiles alongside algebraic formulas, ensuring students with different learning strengths can access the core concept.
- Highlight how diverse historical cultures, including Babylonian, Indian, and Chinese mathematicians, independently discovered these spatial relationships long before Pythagoras.

SOCIAL-EMOTIONAL LEARNING

- Normalize productive struggle when estimating non-perfect square roots, encouraging students to view errors as valuable data points that help refine their mathematical estimates.
- Structure peer feedback sessions to focus on the clarity of mathematical drawings and explanations rather than just the final numerical answer, building collaborative confidence.
- Encourage students to self-monitor their frustration levels during multi-step algebraic isolation and use a step-by-step checklist to regain their footing.

TRAUMA-INFORMED

- Avoid using student heights or personal physical spaces for measurement activities, instead using neutral, pre-made scenarios to maintain emotional safety and privacy.
- Offer choice in how students present their final backyard design portfolio, allowing them to write, speak, or draw their explanations to reduce performance anxiety.
- Ensure all collaborative tasks have clearly defined, low-stakes roles so that no student feels publicly exposed or pressured during group problem-solving.

STAGE 4 Reflect & Revise

Unit Reflection & Communication

Reflection Questions

1. What evidence in the backyard design portfolios showed that students understood how to choose between addition and subtraction when solving for different triangle sides?
2. Where did students get stuck or need more support when translating written real-world scenarios into labeled right triangle diagrams?
3. What would I change next time about how I introduced non-perfect square roots to help students estimate decimal lengths more confidently?

Success Criteria Check

- Check whether students can correctly identify and label the hypotenuse as the side opposite the right angle, regardless of how the triangle is oriented.

- Observe whether students use the correct algebraic steps to isolate a missing leg variable rather than automatically adding the squared values.
- Look for whether students can explain why their calculated real-world distance makes sense by comparing it to the lengths of the other two sides.